THE ADOPTION OF SOME LOCAL STEREOGRAPHIC PROJECTIONS IN THE GENERAL CADASTRE WORK FROM GEOGRAPHICALLY EXTREME LOCALITIES OF ROMANIA

ADOPTAREA UNOR PROIECŢII STEREOGRAFICE LOCALE ÎN LUCRĂRILE DE CADASTRU GENERAL DIN LOCALITĂŢILE GEOGRAFICE EXTREME ALE TERITORIULUI ROMÂNIEI

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Abstract. *The geographical position of Romania is framed by the following extremes represented by these localities: in the North, Horodiştea village in Botoşani County; in the South, the city of Zimnicea în Teleorman County, in the East, the city of Sulina in Tulcea County and the village Beba Veche, in Timiş County, in the West. The relative linear deformations for large scale representations of the stereographic projections -1970 have indicated high and very high values in the case of these four localities, as following: 24 cm/km at Horodiştea, 19 cm/km at Zimnicea, 63 cm/km at Sulina and 58 cm/km at Beba Veche. For each locality was adopted a local secant stereographic plan, different from the unique secant plan – 1970 so as to reduce the distance deformations.*

Key words: stereographic projections -1970, local stereographic projections.

Rezumat. *Poziţia geografică a României este încadrată de către următoarele patru puncte ale zonelor extreme, care sunt reprezentate de următoarele localităţi: satul Horodiştea, judeţul Botoşani, la Nord; oraşul Zimnicea, judeţul Teleorman la Sud; oraşul Sulina, judeţul Tulcea la Est şi comuna Beba Veche, judeţul Timiş, la Vest. Deformaţiile liniare relative determinate pentru reprezentările la scări mari ale proiecţiei Stereografice-1970, au evidenţiat valori mari şi foarte mari, în cazul celor patru localităţi, după cum urmează: 24 cm/km la Horodiştea, 19 cm/km la Zimnicea, 63 cm/km la Sulina şi 58 cm/km la Beba Veche. Pentru fiecare localitate a fost adoptat, câte un plan stereografic secant local, diferit de planul secant unic - 1970, în vederea reducerii deformaţiilor distanţelor.*

Cuvinte cheie: proiecţia stereografică-1970, proiecţie stereografică locală.

INTRODUCTION

The Stereographic projection on unique sectional plane - 1970, complies with the basic cadastral plan drawn up for all administrative units in Romania, where the **relative linear deformations** do not exceed ± **5 cm/km** in the city and ± **15 cm/km** outside the city (Moca V. and Oniga Valeria-Ersilia, 2010; Săvulescu C. and Moldoveanu C., 1997). In the case of territorial units, where the relative linear deformations exceed these values the use of **local stereographical projections** is

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required, leading to reduction and/or cancellation of lengths and surface deformation (Moca, V. and Oniga Ersilia Valeria, 2011).

MATERIAL AND METHOD

The introduction and maintenance of general survey on the administrative-territorial units in the Stereographic projection - 1970 is made based on digital and analog cadastral plans. *The European Terrestrial Reference System-1989* (Dragomir P. I., Rus T. and Dumitru D., 2005; European Terrestrial Reference System, 2009) was adopted since 2009, using satellite positioning technology to the *National Geodetic Network* points. By using the conform reprezentation the maintenance of undistorted representation of angles is ensured, but the lengths and areas are distorted, according to position of points considered to pole projection of Stereographical projection–1970.

Romania's geographical position is delimitated in areas of extreme stateborders, by the following locations (Statistical Yearbook of Romania, 2009): **Horodistea** village, Botosani County in North, **Zimnicea** city, Teleorman county at South, **Sulina** city, Tulcea County in East and **Beba Veche** village, Timis county in west (figure1).

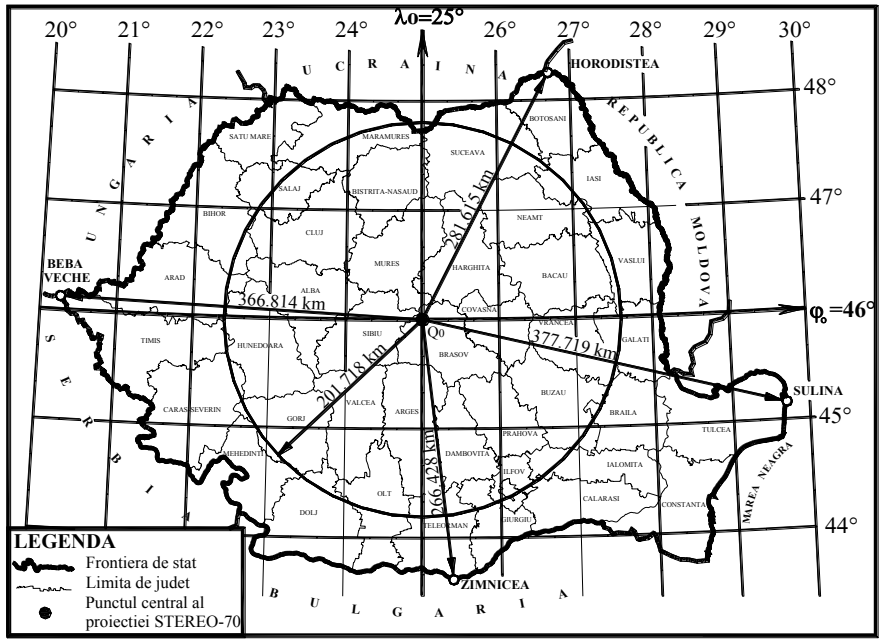

Fig. 1 – Points position of extreme areas of the Romanian territory, relative to the Stereographical projection pole – 1970

Length and surface deformation analysis of extreme points and the bounding trapezoids geodetic mapping, scale 1:5000 was performed according to the spatial variation of the following parameters: linear deformation module (μ), the relative linear deformation (**D**) areolar strain module ($\mathbf{p = \mu^2}$), total areolar strain (**ΔS = S – T**) where: **S** – area of the trapezoid from secant plane -1970; **T** – trapezoid area on the official reference ellipsoid Krasovski – 1940.

For each of the four points/trapezes, a **local stereographical projection system** was adopted, derived from the Stereographical projection – 1970. Next, the following elements were calculated: local secant plane depth, null local deformation radius and coordinates transformation coefficient.

RESULTS AND DISCUSSIONS

For length and surface deformations calculation of the four points (localities) of Romanian territory were used plan stereographical coordinates of the geodetic trapezoids corners, scale 1:5000, with 1'15" latitude and 1 '52 ',5 longitude dimensions. Framing mapping of the four extreme geographical areas included four trapezes, scale 1:5000, with the nomenclature M-35-138-Ab-4-III (Horodistea) K-35-15-BB-3-I (Zimnicea) L-34-65-Yes-3-I (Beba Veche) and L-35-108-Cb-2-I (Sulina).

1. Plan stereographical coordinates - 1970 of the trapezoid corners

Geographic coordinates conversion of the trapezoids corners from Krasovski reference ellipsoid – 1940, to stereographical coordinates – 1970, was made by using formulas with constant coefficients method.

In the first stage were determined stereographical coordinates of "*tangent plane*", parallel to "*the secant plane*", according to difference in **latitude (l)** and **longitude (l)** of the pole projection $\mathbf{Q_0\ (\varphi_0,\ \lambda_0)}$ and the considered point $\mathbf{Pi(\varphi i,\ \lambda i),}$ using constant coefficients of general form (**aij, bij**):

$$X_{tg} < 70 > = (a_{00} + a_{10}f + a_{20}f^2 + a_{30}f^3 + a_{40}f^4 + a_{50}f^5 + a_{60}f^6)\ 1{,}000 + (a_{02} + a_{12}f + a_{22}f^2 + a_{32}f^3 + a_{42}f^4)\ l^2 + (a_{04} + a_{14}f + a_{24}f^2)\ l^4 + (a_{06} + \dots)l^6 \quad [m]$$

$$Y_{tg} < 70 > = (b_{01} + b_{11}f + b_{21}f^2 + b_{31}f^3 + b_{41}f^4 + b_{51}f^5)\ l + (b_{03} + b_{13}f + b_{23}f^2 + b_{33}f^3)\ l^3 + (b_{05} + b_{15}f + \dots)l^5 \quad [m]$$

In the second stage stereographical rectangular coordinates transformation was performed, from the *"tangent plane"* to *"secant plane*" according to the scale reduction factor (**C = 0.999750**), with the following relations:

$$\mathbf{X_{sec} < 70 > = X_{tg} < 70 >\ x\ 0{,}999750}$$
$$\mathbf{Y_{sec} < 70 > = Y_{tg} < 70 >\ x\ 0{,}999750}$$

Stereographical rectangular coordinates of the *"secant plane"* were expressed in the official system, with translated origin by adding the values of **500 000 m** to both coordinates:

$$\mathbf{X < 70 > = X_{sec} < 70 > + 500\ 000{,}000\ m;}$$
$$\mathbf{Y < 70 > = Y_{sec} < 70 > + 500\ 000{,}000\ m}.$$

Geographical ellipsoidal and Stereographical – 1970 coordinates of the four trapeze's corners, scale 1:5000, respectively, of the extreme points (localities) of the Romanian territory are presented in table 1.

Table 1

Geographical and Stereographical-1970 coordinates of the trapezoids corners and extreme points of Romanian territory

No. and point name	Geographic		Stereographic Coordinates (m)	
	φ (° ´ ´´)	λ(° ´ ´´)	X < 70 >	Y < 70 >
	Trapezoid: M-35-138-A-b-4-III			
1 - North West	48 16 15	26 41 15	753 783. 230	625 284. 038
2 - North East	48 16 15	26 43 07.5	751 466. 607	625 334. 063
3 - South West	48 15 00	26 41 15	753 833. 790	627 603. 902
4 - South East	48 15 00	26 43 07.5	751 517. 182	627 654. 853
Horodiştea	**48 15 06**	**26 42 05**	**751 674. 291**	**626 361. 492**
	Trapezoid: K-35-15-B-b-3-I			
1 - North West	43 37 30	25 22 30	236 164. 358	530 271. 954
2 - North East	43 37 30	25 24 22.5	236 176. 482	532 794. 607
3 - South West	48 36 15	25 22 30	233 849. 228	530 282. 634
4 - South East	48 36 15	25 24 22.5	233 861. 354	532 806. 177
Zimnicea	**43 37 07**	**25 23 32**	**235 460. 937**	**531 665. 642**
	Trapezoid: L-34-65-D-a-3-I			
1 - North West	46 07 30	20 15 00	524 849. 404	133 080. 612
2 - North East	46 07 30	20 16 52.5	524 705. 650	135 493. 034
3 - South West	46 06 15	20 15 00	522 536. 449	132 942. 341
4 - South East	46 06 15	20 16 52.5	522 392. 666	135 355. 674
Beba Veche	**46 07 27**	**20 15 44**	**524 700. 549**	**134 018. 612**
	Trapezoid: L-35-108-C-b-2-I			
1 - North West	45 10 00	29 41 15	418 169. 295	868 346. 097
2 - North East	45 10 00	29 43 07.5	418 313. 395	870 800. 270
3 - South West	45 08 45	29 41 15	415 856. 452	868 481. 434
4 - South East	45 08 45	29 43 07.5	416 000. 579	870 936. 511
Sulina	**45 09 36**	**29 41 24**	**417 440. 673**	**868 585. 768**

2. Regional length and surface deformations

Secant plane deformation analysis of the Stereographical projection – 1970, was made for extreme points (localities) that enclose Romania, along the cardinal points. Calculation of deformation modules of length (μ) and relative linear deformations (D) was determined using the following relationships:

$$\mu = D_0 + \frac{L^2}{4R_0^2} \quad \left[km / km\right] \quad si \quad D = (\mu - 1) \cdot 10^5 \quad \left[cm / km\right]$$

where: D_0= - 0.000 250 km / km is the deformation length of projection pole;

$L^2 = (X^2 + Y^2)$ is the distance between the projection pole (Q_0) and the given point (Pi);

R_0 = 6378.956 681 km is the average radius curvature of the reference ellipsoid Krasovski - 1940, for latitude φ_0 = 46 ° 00'00" North.

Relative linear deformations (D) have ranged between **+18.6 cm/km minimum** in the southern extremity (Zimnicea town) and a **maximum** of **+62.7**

cm/km in the eastern extremity (Sulina town), based on size of the distance between analyzed points and the projection pole (table 2).

Table 2

Length and surface deformations of extreme points (localities) of Romanian territory, in Stereographic projection – 1970

Trapezoid and location name	Lenght deformations		Area deformations	
	μ	D	$p = \mu^2$	ΔS = (S-T)
	-	cm/km	-	ha
Horodiştea M-35-138-A-b-4-III	1.000237251	23.7	1.000474558	0.2584
Zimnicea K-35-15-B-b-3-I	1.000186113	18.6	1.000372261	0.2191
Beba Veche L-34-65-D-a-3-I	1.000576671	57.7	1.001153675	0.6441
Sulina L-35-108-C-b-2-I	1.000626553	62.7	1.001253499	0.7195

Total areola deformations (ΔS) of trapezoids areas from secant plane of Stereographical projection–1970, showed values between **2191 m²**, in Zimnicea's trapeze and **7195 m²** in Sulina's trapeze, depending on the distance between the central point of the given trapeze and the projection pole (table 2).

3. Characteristic elements of local Stereographical projections

For the extreme border localities of the Romanian territory: Horodistea, Zimnicea, Beba Veche and Sulina local stereographical projections derived from the Stereographical projection – 1970 were adopted (figure 2).

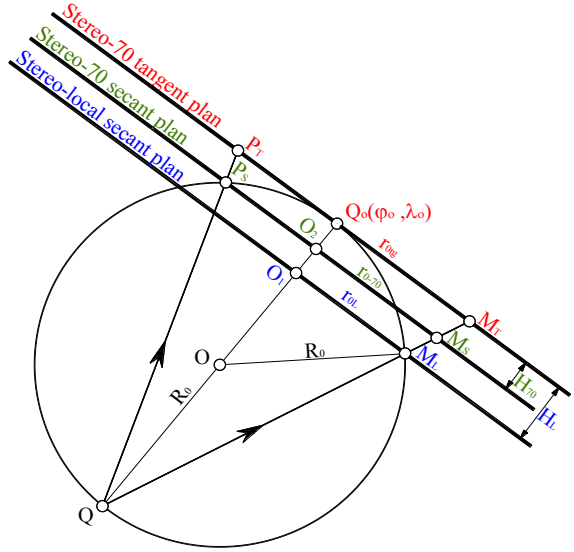

Fig. 2 – The geometric elements (H_L, r_{0L}) and the stereographic local secant plan position, compared to the tangent and unique secant projection plans (H_{70}, $r_{0\text{-}70}$)

To calculate the characteristic elements of the four local stereographical projection systems were used the Stereographical-1970 coordinates of the $\mathbf{M_L}$ points from the middle of four trapezes, scale 1:5000, which enclose extreme geographical areas of the Romanian territory. Thought $\mathbf{M_L}$ **points** "*the secant plane"* of each, local stereographical projection in depth system ($\mathbf{H_L}$) was plotted, parallel to the unique secant plane – 1970 (figure 2).

In the first stage transcalculation parameters of the plan coordinates of stereographical – 1970 system, in the local stereographical system and vice versa were determined based on browsing the following specifical operations:

- **Stereographical-1970 coordinates calculation of the points** ($\mathbf{M_L}$) from the middle of trapezoids, scale 1:5000, based on the location of the four corners (1, 2, 3, 4), with expressions:

$$X_{ML}\langle 70\rangle = \frac{X_1\langle 70\rangle + X_2\langle 70\rangle + X_3\langle 70\rangle + X_4\langle 70\rangle}{4}$$

$$Y_{ML}\langle 70\rangle = \frac{Y_1\langle 70\rangle + Y_2\langle 70\rangle + Y_3\langle 70\rangle + Y_4\langle 70\rangle}{4}$$

- **Stereographical – 1970 coordinates calculation on unique secant plane of the points** ($\mathbf{M_L}$), by cancellation of official translation of the coordinate axes.

$$\mathbf{X < 70 >_{sec} = X < 70 > - 500\,000{,}000\ m}$$
$$\mathbf{Y < 70 >_{sec} = Y < 70 > - 500\,000{,}000\ m}$$

- **Stereographical coordinates calculation on tangent plane of the points** ($\mathbf{M_L}$), with return to scale coefficient (**C '= 1.000250063**).

$$\mathbf{X < 70 >_{tg} = 1{,}000250063} \text{ x } \mathbf{(X < 70 >_{sec})}$$
$$\mathbf{Y < 70 >_{tg} = 1{,}000250063 \text{ x } (Y < 70 >_{sec})}$$

- **Distance calculation between pole projection Q_0 (X_0, Y_0) and the considered point M_L (X <70> tg, Y <70> tg),** from the plane tangent -1970.

$$r_{0tg} \cong r_{0L} = \sqrt{\left(X_{ML}\langle 70\rangle_{tg} - X_0\right)^2 + \left(Y_{ML}\langle 70\rangle_{tg} - Y_0\right)^2}$$

- **Regional strain calculation** along the length of 1 km, in the tangent plane.

$$\Delta r_{tg} = \frac{r_{0tg}^2}{4R_0^2} = \frac{\left(X_{ML}\langle 70\rangle_{tg} - X_0\right)^2 + \left(Y_{ML}\langle 70\rangle_{tg} - Y_0\right)^2}{4R_0^2}$$

- **Reduction coefficient scale calculation (U_{ML})** from the tangent plane in the local secant plane, according to regional deformation.

$$U_{ML} = 1,\ 000\ 000 \text{ km} - \Delta r_{tg}$$

- **Calculation of transformation coefficient (K_{ML})**, coordinates from unique secant-stereographical plan - 1970, in the local stereographical plan and vice versa:

$$K_{ML} = U_{ML} / 0,\ 999\ 750$$

Official and local Stereographical coordinates transcalculation parameters from a system to another system (**U and K**) are presented in table 3.

Table 3

Transcalculation parameters of coordinates from stereographic-1970 system to local stereographic system and vice versa

Place name and nomenclature trapezoid	Transcalculation parameters	Geometrical elements of local stereographical projections		Area deformations	
				Stereo-70 secant plan	Stereo local secant plan
	U/*K*	H_L [m]	r_{oL} [km]	ΔS = (S-T) ha	ΔS = (S-T) ha
Horodiştea M-35-138-A-b-4-III	0.999509/ 0.999759	6260.155	282.537	+ 0.2584	- 0.0004
Zimnicea K-35-15-B-b-3-I	0.999562/ 0.999812	5584.652	266.866	+ 0.2191	- 0.0002
Beba Veche L-34-65-D-a-3-I	0.999174/ 0.999424	10536.338	366.484	+ 0.6441	- 0.0009
Sulina L-35-108-C-b-2-I	0.999118/ 0.999368	11254.287	378.754	+ 0.7195	- 0.0007

In the second stage were determined geometrical elements of the four stereographical projection systems throughout the local secant plane as follows:

- Local secant plane depth (H_L), corresponding to the central point of the trapezoid **(M_L)** of the considered administrative-territorial unit.

$$H_L = 2R_o(1 - K_{ML} \cdot C)\ m$$

- Circle radius of null strain of local secant plane (r_{0L}), expressed in terms of depth to local secant plane (H_L), with the following ratio:

$$r_{0L} = \sqrt{R_0^2 - (R_0 - H_L)^2}\ \ km$$

Geometrical elements (H_L r_{0L}) which were determined in case of using the local Stereographic projection systems are shown in table 3.

In the third stage was made rectangular plan coordinates transformation from secant - 1970 plane to local secant plane and vice versa, as follows:

- Direct transformation of the coordinates from secant plane of Stereographic projection-1970 to secant plane of local stereographical projection, taking into account the translation system of axes and the coefficient (**K**):

$$\begin{cases} X_M\langle L\rangle = X_M\langle 70\rangle \cdot K + 500\,000,000\ m \cdot (1-K) \\ Y_M\langle L\rangle = Y_M\langle 70\rangle \cdot K + 500\,000,000\ m \cdot (1-K) \end{cases}$$

- Inverse transformation of local Stereographic coordinates into Stereographic–1970 coordinates of the same point was verified with the relations:

$$\begin{cases} X_M\langle 70\rangle = X_M\langle L\rangle / K + 500\,000,000\ m \cdot (K-1)/K \\ Y_M\langle 70\rangle = Y_M\langle L\rangle / K + 500\,000,000\ m \cdot (K-1)/K \end{cases}$$

Adopting stereographic projections on local secant plane has determined linear and areolar deformations reduction to negligible values (table 3).

Technical documentations which are prepared for embracing the local stereographic projections derived from Stereographic projection – 1970 are considered unique to each locality or territorial - administrative unit. Their defining elements are: t**he Stereographic-1970 coordinates of the considered area's central point**, **transcalculation coefficient of coordinates, the formulas used for direct and inverse coordinate transformation**.

CONCLUSIONS

1. Analysis of regional lenght deformations has been highlighted depending on the position of the border points in Romania, in relation to the Stereographic - 1970 projection pole, using the following values of relative linear deformations: +**19 cm/km** (Zimnicea), +**24 cm/km** (Horodistea), +**58 cm/km** (Beba Veche) and +**63 cm/km** (Sulina);

2. Total areolar surface deformations of trapezoid mapping areas of border localities, showed the same spatial variation in length, as evidenced by the values: **+2191 m^2** (Zimnicea) +**2584 m^2** (Horodistea) +**6441 m^2** (Beba Veche) and +**7195 m^2** (Sulina);

3. Projection systems adopted on a local Stereographic secant plane, parallel to the secant plane of Stereographical projection - 1970, has significantly reduced lengths and surface deformation to negligible values, which ensures high precision calculation of surfaces in the work input and maintenance of the general cadastre.

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